**Problem 6**

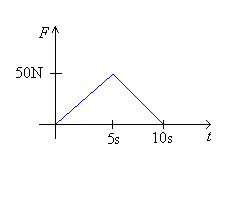
A pitcher throws a baseball toward the batter in the negative x direction with a speed of 26 m/s. The batter strikes the ball with a force that varies with time according to F(t) = 3000t Newtons. So at what time does the ball start to move in the positive x direction?

Assume that the baseball has a mass of 0.25 kg.

Use the impulse-momentum equation,



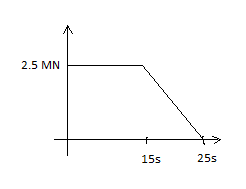
7. Suppose you push on a 50 kg box according to the following changing force. What will be its velocity after 10s (in m/s)?



Using the IM equation,



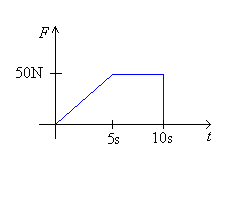
**Question 1. A rocket’s engine provide the following force on a rocket of mass m = 1740kg. If its initial speed were v0 = 8 500 m/s, what is its final speed?**

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**Impulse is given by area under curve, which is J = (2.5×106)(25+15)/2 = 50×106 N∙s. Plugging into the impulse-momentum equation:**

****

3. Suppose you push on a 45 kg shopping cart according to the following changing force. What will be its velocity after 10s? C



The velocity can be determined from the impulse-momentum equation. We have:



**Example**

Suppose a baseball (0.15 kg) comes towards you at a speed of 40m/s, and upon hitting it with your bat, it emerges with a speed of 50m/s. What is the impulse delivered to the ball by the bat?

According to the equation above, we have,



If the impact between ball and bat lasts 0.12s, then what average force is delivered by the bat?

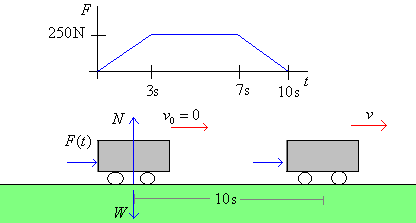


therefore,



**Example**

Suppose you push on a 30 kg shopping cart according to the following changing force. What will be the velocity after 10s?



To answer we use the impulse-momentum equation – surprise!



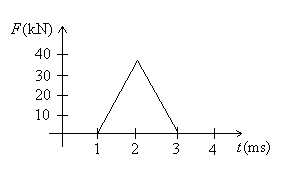
The impulse in the y-direction cancels out because the normal force and gravity cancel out. And thus we simply have,



Now the integral is just the area under the F-t curve, which is, according to the graph above, J = 1750N·s. So we have,



3. Suppose that a pitcher throws a baseball toward the batter in the negative x direction with a speed of 40 m/s. The batter swings and makes contact with ball, which experiences a force that varies over time according to the graph shown below. What is the final velocity of the ball? Assume that the baseball has a mass of 0.25 kg.

****

Use the impulse-momentum equation,



Now the impulse is given by the area of the F-t curve, which is J = (1/2)(0.002)(40000) = 40N·s. And the initial velocity is -40m/s. Therefore the final velocity is:



4. It is well known that bullets and other missiles fired at Superman simply bounce off his chest. Suppose that a gangster sprays Superman's chest with 8 g bullets at the rate of 210 bullets/min, and the speed of each bullet is 300 m/s. Suppose too that the bullets rebound straight back with no change in speed. What is the magnitude of the average force on Superman's chest from the stream of bullets?

Using the impulse-momentum equation…

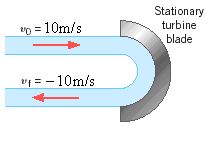


where N is the number of bullets hitting Superman, m is their mass, Δv is their change in velocity, and Δt is the duration of the process. So filling in the numbers N = 210, m = 0.008kg, Δv = 300-(-300) = 600m/s, and Δt = 60s we get:



6. A stream of water strikes a stationary turbine blade horizontally, as the drawing illustrates. The incident water stream has a velocity of 10.0 m/s, while the exiting water stream has a velocity of -10.0 m/s. The mass of water per second that strikes the blade is 20.0 kg/s. Find the magnitude of the average force exerted on the water by the blade.

(could just make this a fire hose hitting someone)



Use the IM equation – over span of 1s.



**Problem 2**

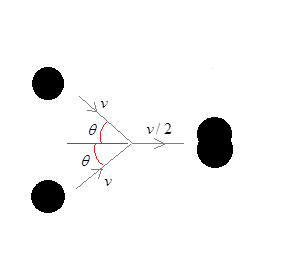
Yoda, who has a mass of 25kg, is wearing skates on an iced over pond. He uses the ‘Force’ to horizontally hurl a bowling ball (with mass 5kg) in the positive x direction, at a speed of 50m/s. What is his resulting velocity?

We have,



5. After a completely inelastic collision (meaning objects stick together after collision), two objects of the same mass and same initial speed are found to move away together at 1/2 their initial speed. Find the angle between the initial velocities of the objects.

Situation looks like this:

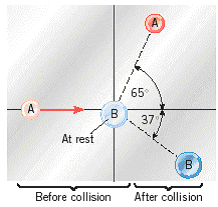


Conservation of momentum equation reads,



and so the angle *between* the two masses is 120◦.

3. The drawing shows a collision between two pucks on an air-hockey table. Puck A has a mass of 2 kg and is moving along the x axis with a velocity of 5m/s. It makes a collision with puck B, which has a mass of 4 kg and is initially at rest. The collision is not head-on. After the collision, the two pucks fly apart with the angles shown in the drawing. Find the speed of (a) puck A and (b) puck B.



Let vA be the speed of A, and vB be the speed of B. Conservation of momentum in the x direction gives,



And in the y-direction, conservation of momentum says,



Plugging this back into the first equation gives,



And therefore vB is equal to:



**Example: car crash**

Suppose you get into a car accident with another car, and the dashboard decelerates you from 20m/s to 0 in a matter of 0.4s, what force do you experience, assuming your mass is 70kg?

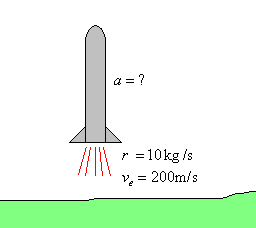
Again, use IM equation.



Note that the impulse delivered by N and mg presumably cancel out again since those forces are equal (since there is no acceleration in the y-direction).

**Example: Rocket**

Suppose a rocket (M = 150kg) ejects fuel at a rate r = 10 kg/s with a speed of ve = 200m/s (relative to the rocket). What acceleration will this give the rocket? What initial force will this exert on the rocket?



To figure this out let’s use the impulse momentum equation on the rocket + fuel. The force with which the rocket ejects the fuel (and by N3L the force which the fuel exerts on the rocket) is an internal force. The only external force is that of gravity. So let’s apply the impulse momentum equation to the rocket + fuel over a short time interval Δt after launch. Then we have,



So we see that the force that ejecting the fuel provides is:



The acceleration of the rocket would be, making the approximation in the first line of the previous paragraph.



**Example: Rocket again**

Suppose a rocket ejects fuel mass with rate r, at velocity ve. How fast will it be going as a function of time? What will be its final velocity?



Now, the mass of the rocket is changing with time since part of it is being continuously ejected as fuel .So mrocket(t) = m0 – rt, where m0 is its initial mass. Plug this in and write a = dv/dt,

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Assuming an initial velocity of 0, then the velocity of the rocket as a function of time will be:



The final velocity will be achieved when the rocket runs out of fuel, at tfinal = mfuel/r.

**Example:**

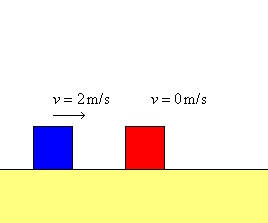
Suppose the rocket in the first example has a store of rocket fuel = 120kg, while the rest of the rocket has a mass of 30kg. And suppose that it ejects mass at a rate of r = 10kg/s, at a speed of 200m/s. What will be the final velocity of the rocket?

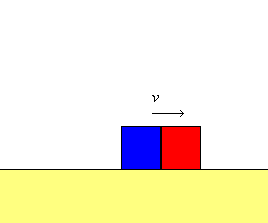
All the fuel will be ejected after tfinal = mfuel/r = 120/10 = 12s. So filling these into the formula,



**Example: collisions**

Suppose you slide a blue crate (mblue = 20kg) along the floor toward another red one (mred = 30kg). Suppose that just before the collision the blue one has a velocity 2m/s to the right and the red one is stationary. Also suppose that just after the collision, they emerge moving together at a certain velocity v. What is this velocity? Assume the collision lasts 0.2s, and that the coefficient of kinetic friction is μk = 0.1.





We can use the impulse-momentum equation. Note that the external impulses would be due to gravity, the normal forces, and friction. Gravity and the normal force cancel, leaving only friction.



Plugging in the values this turns into:



So the blocks will move off together with a speed of 1.66 m/s.

**Example: collision between pool balls**

Suppose you are playing pool, and you strike a red ball giving it a speed of 2m/s, and it hits another stationary yellow ball. If the red one rebounds backwards with a speed of 0.5m/s, what is the speed of the yellow one? Assume that friction is negligible.

The gravitational and normal forces cancel, so their impulses do as well. Additionally, assuming that friction is non-existent or negligible (as is the case when objects are rolling), then its impulse is zero too. In this case we have



So the yellow one goes forward with a speed of 2.5m/s.

**Example: 2 dimensions**

Consider that you (m = 70kg) are skating along with a speed of 5m/s east, when suddenly someone throws a bowling ball (m = 8kg) at you with a velocity of 5m/s north. After you catch it, what will be your new speed and direction? Again, assume friction is negligible.

We will apply the impulse momentum equation again. Like last time, since the normal forces and gravity cancel, their impulses will be 0. And if friction is negligible, then so too will its impulse. So we have,



Therefore our speed will be:



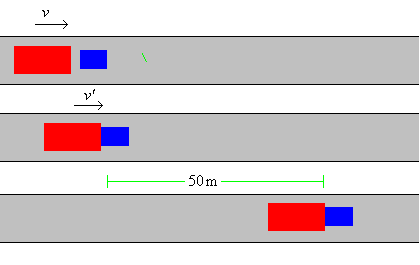
and the angle would be:

 North of East.

**Example: Traffic collision**

Suppose you’re CSI guy. You come to an car accident scene. Apparently a truck (m = 5000kg) ran into a car (1000kg) stopped at a red light. As a result of the collision, both cars skidded together along the road (μk = 0.4) and came to rest after 50m. The speed limit on the rode is 45mph. Your job is to determine whether or not the truck driver was speeding at the time of the collision.

The top-down view is this:



The truck is traveling with velocity v when it collides with the car. The two emerge from the collision with velocity v′ and come to rest 50m later. To determine v, we will apply the impulse momentum equation to the collision and then the WE equation to the sliding. Apropos the first, the normal and gravitational forces will cancel and so their impulses will too. Friction is definitely present this time, and so it will exert an impulse which will keep momentum from being conserved. Nonetheless, we will assume that the impulse delivered by friction is small compared to the total momenta of the cars before and after the collision (this is a safe assumption), and so momentum will be approximately conserved. So assuming this, and using the impulse-momentum equation during the collision we have,



and then during the sliding, the only force acting on the two is that of friction (N and mg cancel out). So applying the WE equation,



Now plug this into the conservation of momentum equation,



Converting to mph we have v = 23.8(2.25) = 53.5 mph. So the truck was indeed speeding.

**Example: Elastic collision**

A proton (mp = 1.67×10-27kg) hurtles with speed vp = 106m/s towards a stationary α particle (i.e., a He2 nucleus) (mα ≈ 4mp). What are the velocities v′p, v′α of both particles after the collision?

Well, the interaction between the two particles is electric force. We don’t know what this force is, but we don’t need to (you’ll learn more than you ever wanted to know about it next semester). All we need to know is that it is a conservative force, therefore momentum and kinetic energy will be conserved. Given this, we can solve for the final velocities. The impulse momentum-equation yields,



Applying the Work-Energy equation to the collision too, and using the fact that there are no non-conservative forces acting on the objects.



These two equations we must solve for v′p and v′α, the velocities of the proton and electron after the collision. We’ll do this in the usual fashion – just solve for one variable using the momentum equation and plug it into the work-energy equation,



and so we have finally,

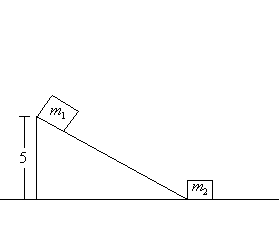


and the velocity of the alpha particle will be, from the impulse-momentum equation:



so we see that the proton bounces back the way it came (since its velocity is negative), and it pushes the alpha particle to the right.

7. Block 1 (mass = 1kg) is released from rest at the top of a frictionless inclined plane, 5m above the floor. It collides with block 2 (mass 2.3kg) located at the base of the incline. If the resulting collision is completely *elastic*, what is the velocity of block 2 after the collision?



First we need to determine the velocity of block 1 right before the collision. We can use WE equation,



Then applying the impulse-momentum equation to the collision we see that since there is no external force acting on the two blocks, their total momentum is conserved so we have:



and applying the work-energy equation to the collision, since there is no work done on or between the blocks during the collision (since it is elastic), energy is conserved during the collision as well. So we have:



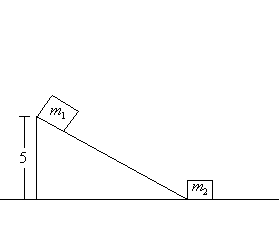
So these are our two equations. We want v′2. So let’s solve for v1 using the first equation and plug it into the second to get v′2. We have:



and plugging into the second,



1. Block 1 (mass = 1kg) is released from rest at the top of a frictionless inclined plane, 5m above the floor. It collides with block 2 (mass 2kg) located at the base of the incline. If the resulting collision is completely inelastic, i.e., they stick together, how far do the two blocks travel before coming to rest if the coefficient of kinetic friction of the floor is 0.1?





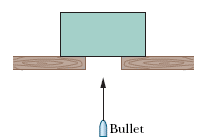
Then, using conservation of momentum, we can determine the velocity of the pair after the collision.



And finally, the distance that they travel is, using the work-energy equation,



4. In the figure, a bullet with mass *m*1 = 0.015kg moving directly upward with speed *v*1*i* = 300m/s strikes and passes through the center of mass of a block with mass *m*2 = 2.4kg which is initially at rest. The bullet emerges from the block moving directly upward and has slowed to a speed *v*1*f*. = 120m/s. To what maximum height does the block then rise above its initial position?



Apply conservation of momentum to the collision to get velocity of block after collision. We have:



Conservation of energy on block gives:



5. A 1000kg car has a velocity of v = 20m/s. How fast would a person, mass m = 70kg, have to run in order to have the same momentum? E



In order for the person to have the same momentum we would need,



6. You are pointing a rifle at a target. Upon firing the rifle, the bullet (mb = 0.015kg) exits the barrel of the gun in 5ms, with a speed of 300m/s. What recoil force does the bullet/gun exert on you? A

Using the impulse-momentum equation we can determine the force exerted on the bullet.



7. The 1000kg car traveling at 20m/s hits a very heavy pedestrian (m = 300kg). Supposing the pedestrian sticks to the car, what is the new velocity of the car? C



8. A Georgia Bulldog football player (mG = 100kg) is running with the football with a velocity of 7m/s East. A Florida Gators player (mF = 75kg) is running towards him from the opposite direction. What speed must he have so that when he tackles the Georgia player (and sticks to him) they both immediately come to rest? B



9. Football player 1 (m = 120kg) is running with speed of 5m/s East. Football player 2 (m = 90kg) is running 7m/s North. When Football player 2 tackes Football player 1, they stick together and emerge from the collision moving at a certain angle θ North of East. What is this angle? E



So the magnitude and direction of the velocity is:



10. Suppose you fire a bullet (m = 0.015kg) with a speed of 300m/s. It collides with and gets stuck in a block of wood (m = 0.250kg) that is initially stationary. . After the collision the bullet/block slides along the floor which has a coefficient of kinetic fricion μk = 0.30. How far does the bullet/block go before coming to rest? D

Applying conservation of momentum to the collision we can determine the speed of the bullet/block after the collision.



and then we can use the WE equation to get the distance traveled before coming to rest.



or

The acceleration will be ax = μkFN/m = μkg = 2.94 m/s2. And so the time it takes to stop will be:

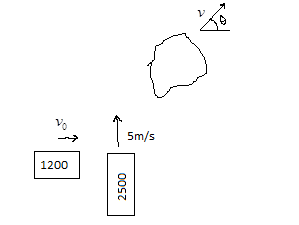


and so the distance travelled will be:



**Question 2.** The stoplight had just changed and a 2500kg Cadillac had entered the intersection, heading north at 5m/s , when it was struck by a 1200kg eastbound Volkswagen. The cars stuck together and slid to a halt, leaving skid marks angled 30° north of east.

Situation looks like this:



Applying conservation of momentum in x and y directions:





Filling this into the top equation we get v0:



**Question 1.** A m = 1500kg car is rolling along the road with speed v = 12m/s. Then a cow (m = 800kg) jumps into the car. What will be its new speed? How much energy is lost in the collision?

Conservation of momentum requires:



Energy lost is:



**Question 2.**  If you throw a 50g bouncy ball with speed v = 30m/s at a stationary 450g baseball, what velocities will the balls have upon emerging from the collision?



Conservation of kinetic energy requires:



Now solve for v1´ in the top equation and plug it into the second:



Plugging this back into the first (conservation of momentum) equation we get:



**Question 10.** A proton is traveling to the right at 4×106 m/s. It has a head-on perfectly elastic collision with a stationary carbon atom. The mass of the carbon atom is 12 times the mass of the proton. What are the velocities of the two objects after the collision?

First we will apply the impulse momentum equation to the collision: pi + J = pf. Since J = 0 for collisions, we have pi = pf. So,



and the work-energy equation looks like Ei + Wn.c. = Ef. For elastic collisions we have Wn.c. = 0, so Ei = Ef, and so:



Solving for v´p in the first equation and plugging it into the second we have:



and for the proton we have:

